### 1.2.2 EXERCISES

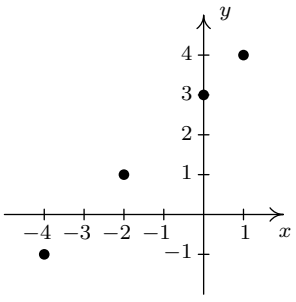
To see all of the help resources associated with this section, click OSttS Chapter 1a.
In Exercises 1 - 20, graph the given relation.
For help with these exercises click on one or more of the resources below:

- Graphing relations
- Graphing linear inequalities in one variable in the plane

1. $\{(-3,9),\ (-2,4),\ (-1,1),\ (0,0),\ (1,1),\ (2,4),\ (3,9)\}$
2. $\{(-2,0),\ (-1,1),\ (-1,-1),\ (0,2),\ (0,-2),\ (1,3),\ (1,-3)\}$
3. $\{(m,2m)\mid m=0,\pm1,\pm2\}$
4. $\left\{\left(\frac{6}{k},k\right)\mid k=\pm1,\pm2,\pm3,\pm4,\pm5,\pm6\right\}$
5. $\left\{\left(n,4-n^2\right)\mid n=0,\pm1,\pm2\right\}$
6. $\left\{\left(\sqrt{j},j\right)\mid j=0,1,4,9\right\}$
7. $\{(x,-2)\mid x>-4\}$
8. $\{(x,3)\mid x\leq 4\}$
9. $\{(-1,y)\mid y>1\}$
10. $\{(2,y)\mid y\leq 5\}$
11. $\{(-2,y)\mid -3<y\leq 4\}$
12. $\{(3,y)\mid -4\leq y<3\}$
13. $\{(x,2)\mid -2\leq x<3\}$
14. $\{(x,-3)\mid -4<x\leq 4\}$
15. $\{(x,y)\mid x>-2\}$
16. $\{(x,y)\mid x\leq 3\}$
17. $\{(x,y)\mid y<4\}$
18. $\{(x,y)\mid x\leq 3,\ y<2\}$
19. $\{(x,y)\mid x>0,\ y<4\}$
20. $\{(x,y)\mid -\sqrt{2}\leq x\leq \frac{2}{3},\ \pi<y\leq \frac{9}{2}\}$

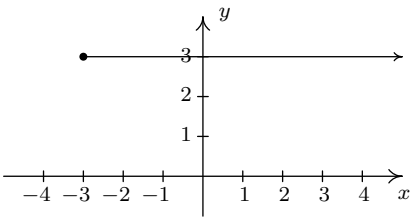
In Exercises 21 - 30, describe the given relation using either the roster or set-builder method.

21.



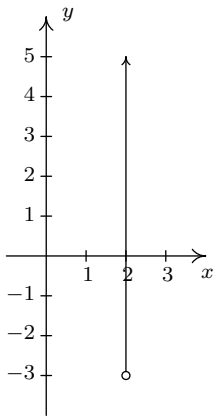

Relation $A$

22.



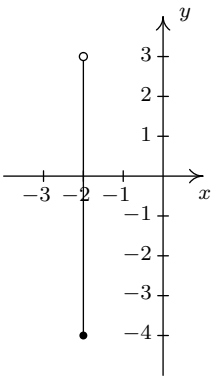

Relation $B$

23.



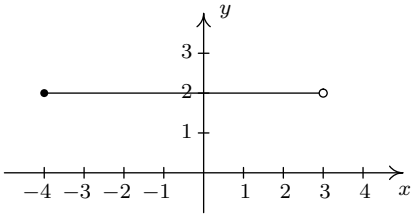

Relation $C$

24.



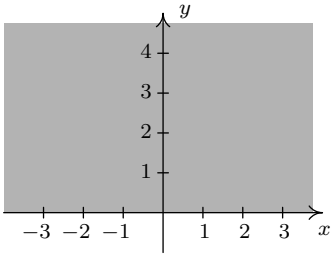

Relation $D$

25.



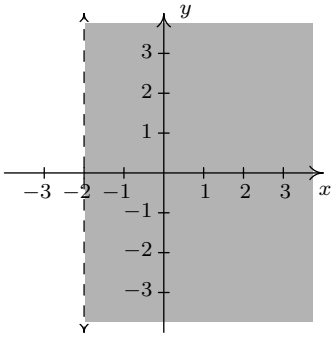

Relation $E$

26.



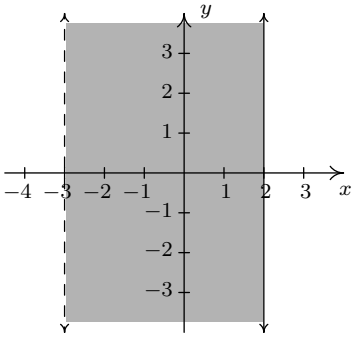

Relation $F$

27.



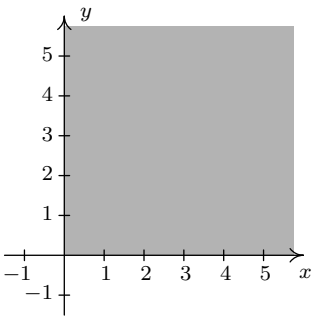

Relation $G$

28.



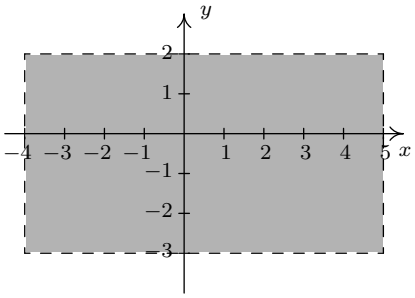

Relation $H$

29.




Relation $I$

30.




Relation $J$

In Exercises 31 - 36, graph the given line. For help, click graphing horizontal and vertical lines.

31. $x = -2$

32. $x = 3$

33. $y = 3$

34. $y = -2$

35. $x = 0$

36. $y = 0$

Some relations are fairly easy to describe in words or with the roster method but are rather difficult, if not impossible, to graph. Discuss with your classmates how you might graph the relations given in Exercises 37 - 40. Please note that in the notation below we are using the ellipsis, ..., to denote that the list does not end, but rather, continues to follow the established pattern indefinitely. For the relations in Exercises 37 and 38, give two examples of points which belong to the relation and two points which do not belong to the relation.

37. $\{(x, y) \,|\, x \text{ is an odd integer, and } y \text{ is an even integer.}\}$

38. $\{(x, 1) \,|\, x \text{ is an irrational number }\}$

39. $\{(1, 0), (2, 1), (4, 2), (8, 3), (16, 4), (32, 5), \ldots\}$

40. $\{\ldots, (-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), \ldots\}$

For each equation given in Exercises 41 - 52:

- Find the $x$- and $y$-intercept(s) of the graph, if any exist.
- Follow the procedure in Example 1.2.3 to create a table of sample points on the graph of the equation.
- Plot the sample points and create a rough sketch of the graph of the equation.
- Test for symmetry. If the equation appears to fail any of the symmetry tests, find a point on the graph of the equation whose reflection fails to be on the graph as was done at the end of Example 1.2.4

For help with these exercises, click on one of more of the resources below:

- Finding intercepts and graphing
- Graphing equations by plotting points
- Testing an equation to see if its graph possesses symmetry

41. $y = x^2 + 1$
42. $y = x^2 - 2x - 8$
43. $y = x^3 - x$
44. $y = \frac{x^3}{4} - 3x$
45. $y = \sqrt{x - 2}$
46. $y = 2\sqrt{x + 4} - 2$
47. $3x - y = 7$
48. $3x - 2y = 10$
49. $(x + 2)^2 + y^2 = 16$
50. $x^2 - y^2 = 1$
51. $4y^2 - 9x^2 = 36$
52. $x^3y = -4$

The procedures which we have outlined in the Examples of this section and used in Exercises 41 - 52 all rely on the fact that the equations were "well-behaved". Not everything in Mathematics is quite so tame, as the following equations will show you. Discuss with your classmates how you might approach graphing the equations given in Exercises 53 - 56. What difficulties arise when trying to apply the various tests and procedures given in this section? For more information, including pictures of the curves, each curve name is a link to its page at www.wikipedia.org. For a much longer list of fascinating curves, click here.

53. $x^3 + y^3 - 3xy = 0$ Folium of Descartes
54. $x^4 = x^2 + y^2$ Kampyle of Eudoxus
55. $y^2 = x^3 + 3x^2$ Tschirnhausen cubic
56. $(x^2 + y^2)^2 = x^3 + y^3$ Crooked egg
57. With the help of your classmates, find examples of equations whose graphs possess

    - symmetry about the $x$-axis only
    - symmetry about the $y$-axis only
    - symmetry about the origin only
    - symmetry about the $x$-axis, $y$-axis, and origin

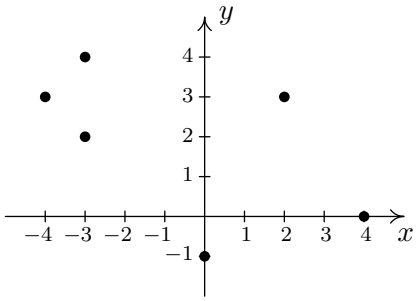
    Can you find an example of an equation whose graph possesses exactly *two* of the symmetries listed above? Why or why not?

**Checkpoint Quiz 1.2**

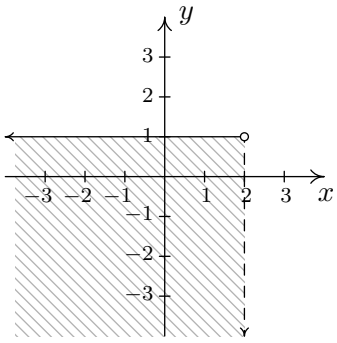
1. Sketch the following relations:

   (a) $\{(-3, 2), (-1, 0), (0, -2), (1, 3)\}$
   (b) $\left\{\left(n, \frac{6}{n}\right) : n = \pm 1, \pm 2, \pm 3)\right\}$
   (c) $\{(x, -2) : x > 1\}$
   (d) $\{(x, y) : x > 1,\ y \leq 2\}$
   (e) $\{(x, y) : x > 1,\ -1 < y \leq 2\}$

2. Describe the following relations using the roster method.




(a) The graph of relation A




(b) The graph of relation $B$

3. Consider the equation: $(x - 1)^2 - y^2 = 25$

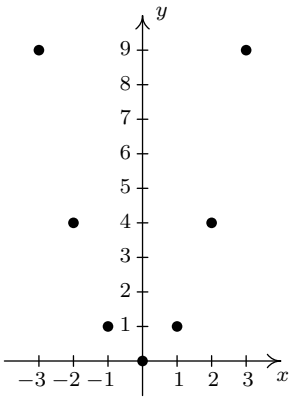
   - Find the $x$- and $y$-intercept(s) of the graph, if any exist.
   - Create a table of sample points on the graph of the equation.
   - Plot the sample points and create a rough sketch of the graph of the equation.
   - Test for symmetry. If the equation appears to fail any of the symmetry tests, find a point on the graph of the equation whose reflection fails to be on the graph.

For worked out solutions to this quiz, click the links below:

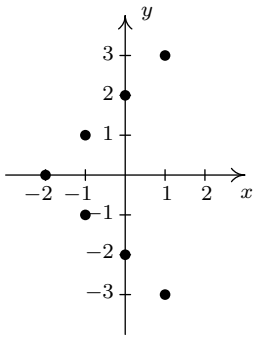
- Quiz Solution Part 1
- Quiz Solution Part 2
- Quiz Solution Part 3
- Quiz Solution Part 4

### 1.2.3 Answers

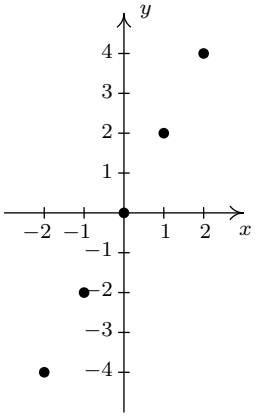
1.



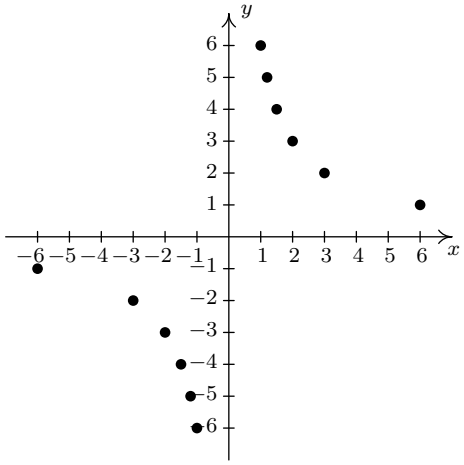

2.



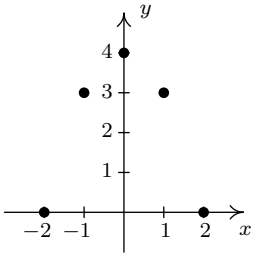

3.



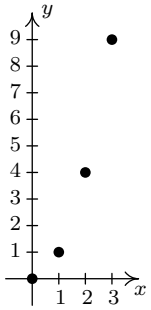

4.



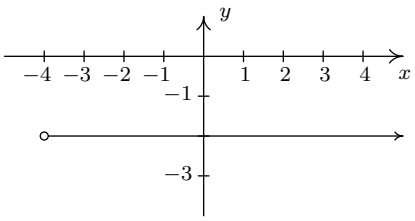

5.



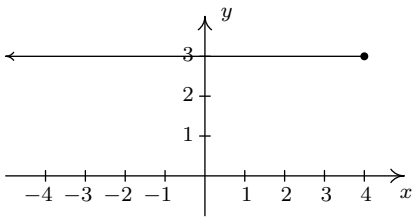

6.



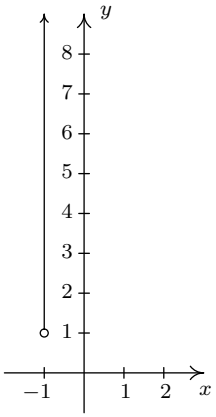
7.


y
−4 −3 −2 −1 1 2 3 4 x
−1
−3

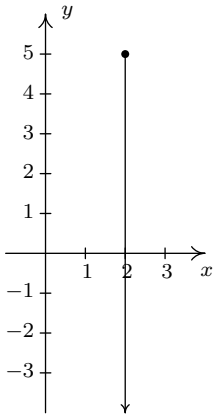
8.


y
3
2
1
−4 −3 −2 −1 1 2 3 4 x

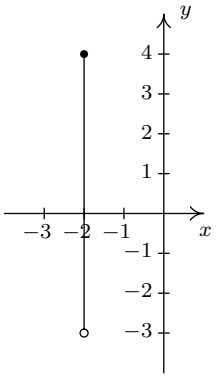
9.


y
8
7
6
5
4
3
2
1
−1 1 2 x

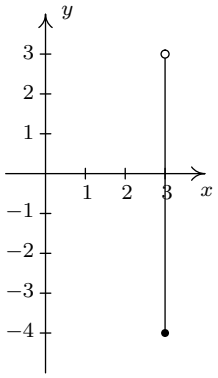
10.


y
5
4
3
2
1
1 2 3 x
−1
−2
−3

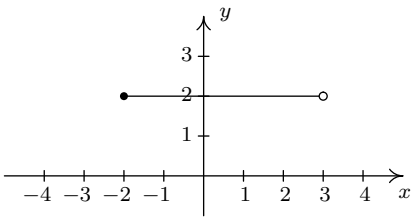
11.


y
4
3
2
1
−3 −2 −1 x
−1
−2
−3

12.


y
3
2
1
1 2 3 x
−1
−2
−3
−4

13.

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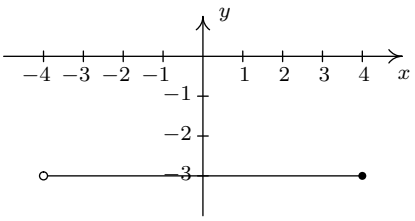


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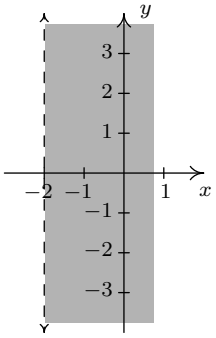
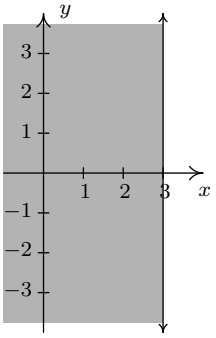



16.

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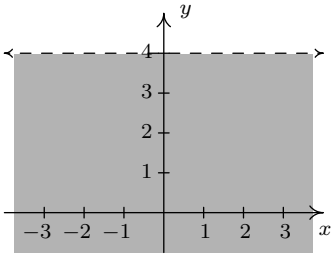
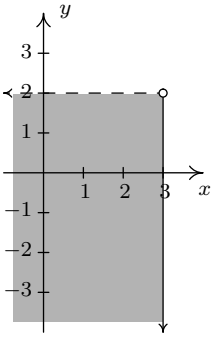



18.

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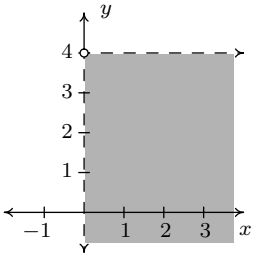
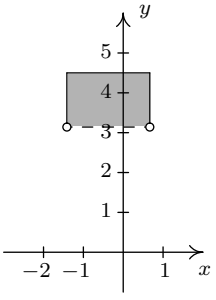



20.



21. $A = \{(-4,-1),(-2,1),(0,3),(1,4)\}$

22. $B = \{(x,3) \mid x \geq -3\}$

23. $C = \{(2,y) \mid y > -3\}$

24. $D = \{(-2,y) \mid -4 \leq y < 3\}$

25. $E = \{(x,2) \mid -4 < x \leq 3\}$

26. $F = \{(x,y) \mid y \geq 0\}$

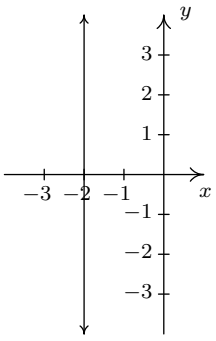
27. $G = \{(x,y) \mid x > -2\}$

28. $H = \{(x,y) \mid -3 < x \leq 2\}$

29. $I = \{(x,y) \mid x \geq 0, y \geq 0\}$

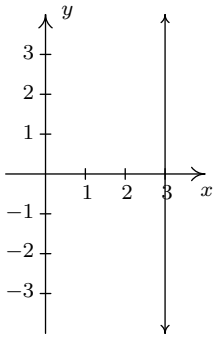
30. $J = \{(x,y) \mid -4 < x < 5,\ -3 < y < 2\}$

31.



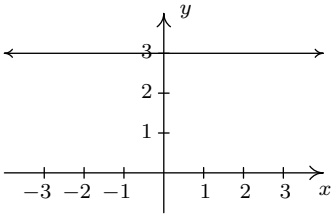

The line $x = -2$

32.



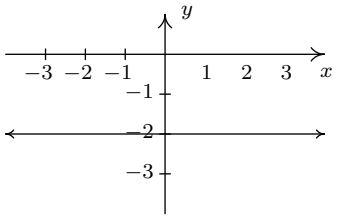

The line $x = 3$

33.



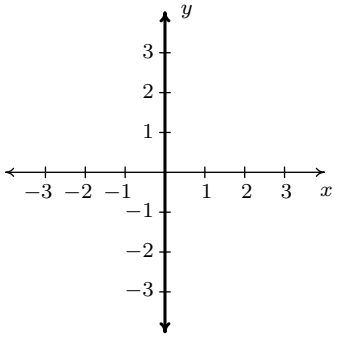

The line $y = 3$

34.



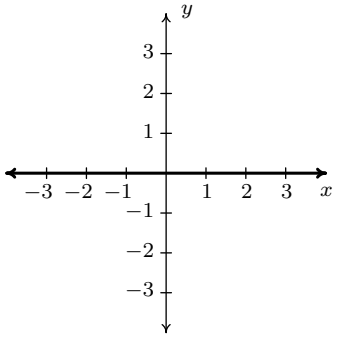

The line $y = -2$

35.




The line $x = 0$ is the $y$-axis

36.



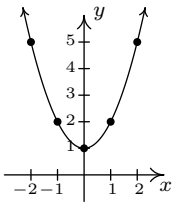

The line $y = 0$ is the $x$-axis

41. $y = x^2 + 1$

The graph has no $x$-intercepts

$y$-intercept: $(0, 1)$

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| $-2$ | 5 | $(-2, 5)$ |
| $-1$ | 2 | $(-1, 2)$ |
| 0 | 1 | $(0, 1)$ |
| 1 | 2 | $(1, 2)$ |
| 2 | 5 | $(2, 5)$ |




The graph is not symmetric about the $x$-axis (e.g. $(2, 5)$ is on the graph but $(2, -5)$ is not)

The graph is symmetric about the $y$-axis

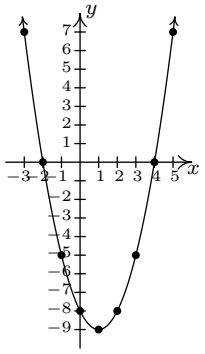
The graph is not symmetric about the origin (e.g. $(2, 5)$ is on the graph but $(-2, -5)$ is not)

42. $y = x^2 - 2x - 8$

$x$-intercepts: $(4, 0)$, $(-2, 0)$

$y$-intercept: $(0, -8)$

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| $-3$ | 7 | $(-3, 7)$ |
| $-2$ | 0 | $(-2, 0)$ |
| $-1$ | $-5$ | $(-1, -5)$ |
| 0 | $-8$ | $(0, -8)$ |
| 1 | $-9$ | $(1, -9)$ |
| 2 | $-8$ | $(2, -8)$ |
| 3 | $-5$ | $(3, -5)$ |
| 4 | 0 | $(4, 0)$ |
| 5 | 7 | $(5, 7)$ |




The graph is not symmetric about the $x$-axis (e.g. $(-3, 7)$ is on the graph but $(-3, -7)$ is not)

The graph is not symmetric about the $y$-axis (e.g. $(-3, 7)$ is on the graph but $(3, 7)$ is not)

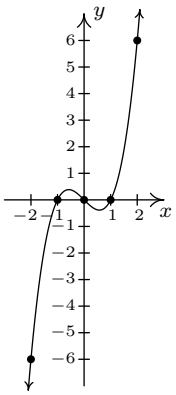
The graph is not symmetric about the origin (e.g. $(-3, 7)$ is on the graph but $(3, -7)$ is not)

43. $y = x^3 - x$

$x$-intercepts: $(-1, 0), (0, 0), (1, 0)$

$y$-intercept: $(0, 0)$

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| $-2$ | $-6$ | $(-2, -6)$ |
| $-1$ | $0$ | $(-1, 0)$ |
| $0$ | $0$ | $(0, 0)$ |
| $1$ | $0$ | $(1, 0)$ |
| $2$ | $6$ | $(2, 6)$ |




The graph is not symmetric about the $x$-axis. (e.g. $(2, 6)$ is on the graph but $(2, -6)$ is not)

The graph is not symmetric about the $y$-axis. (e.g. $(2, 6)$ is on the graph but $(-2, 6)$ is not)

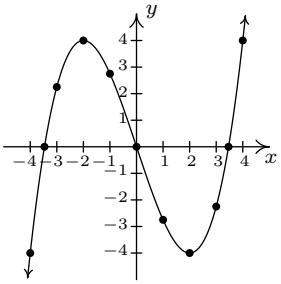
The graph is symmetric about the origin.

44. $y = \frac{x^3}{4} - 3x$

$x$-intercepts: $\left(\pm 2\sqrt{3}, 0\right), (0, 0)$

$y$-intercept: $(0, 0)$

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| $-4$ | $-4$ | $(-4, -4)$ |
| $-3$ | $\frac{9}{4}$ | $\left(-3, \frac{9}{4}\right)$ |
| $-2$ | $4$ | $(-2, 4)$ |
| $-1$ | $\frac{11}{4}$ | $\left(-1, \frac{11}{4}\right)$ |
| $0$ | $0$ | $(0, 0)$ |
| $1$ | $-\frac{11}{4}$ | $\left(1, -\frac{11}{4}\right)$ |
| $2$ | $-4$ | $(2, -4)$ |
| $3$ | $-\frac{9}{4}$ | $\left(3, -\frac{9}{4}\right)$ |
| $4$ | $4$ | $(4, 4)$ |




The graph is not symmetric about the $x$-axis (e.g. $(-4, -4)$ is on the graph but $(-4, 4)$ is not)

The graph is not symmetric about the $y$-axis (e.g. $(-4, -4)$ is on the graph but $(4, -4)$ is not)

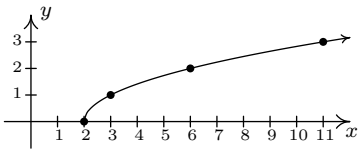
The graph is symmetric about the origin

45. $y = \sqrt{x-2}$

$x$-intercept: $(2, 0)$

The graph has no $y$-intercepts

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| 2 | 0 | $(2, 0)$ |
| 3 | 1 | $(3, 1)$ |
| 6 | 2 | $(6, 2)$ |
| 11 | 3 | $(11, 3)$ |




The graph is not symmetric about the $x$-axis (e.g. $(3, 1)$ is on the graph but $(3, -1)$ is not)

The graph is not symmetric about the $y$-axis (e.g. $(3, 1)$ is on the graph but $(-3, 1)$ is not)

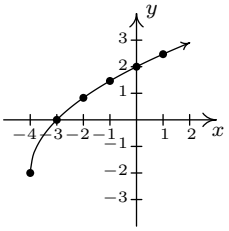
The graph is not symmetric about the origin (e.g. $(3, 1)$ is on the graph but $(-3, -1)$ is not)

46. $y = 2\sqrt{x+4} - 2$

$x$-intercept: $(-3, 0)$

$y$-intercept: $(0, 2)$

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| $-4$ | $-2$ | $(-4, -2)$ |
| $-3$ | 0 | $(-3, 0)$ |
| $-2$ | $2\sqrt{2} - 2$ | $\left(-2, \sqrt{2} - 2\right)$ |
| $-1$ | $2\sqrt{3} - 2$ | $\left(-2, \sqrt{3} - 2\right)$ |
| 0 | 2 | $(0, 2)$ |
| 1 | $2\sqrt{5} - 2$ | $\left(-2, \sqrt{5} - 2\right)$ |




The graph is not symmetric about the $x$-axis (e.g. $(-4, -2)$ is on the graph but $(-4, 2)$ is not)

The graph is not symmetric about the $y$-axis (e.g. $(-4, -2)$ is on the graph but $(4, -2)$ is not)

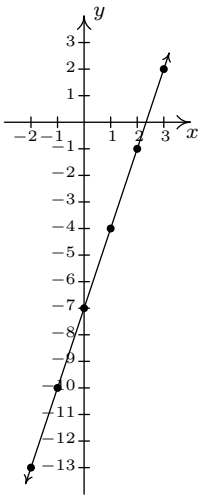
The graph is not symmetric about the origin (e.g. $(-4, -2)$ is on the graph but $(4, 2)$ is not)

47. $3x - y = 7$
Re-write as: $y = 3x - 7$.

$x$-intercept: $(\frac{7}{3}, 0)$

$y$-intercept: $(0, -7)$

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| $-2$ | $-13$ | $(-2, -13)$ |
| $-1$ | $-10$ | $(-1, -10)$ |
| $0$ | $-7$ | $(0, -7)$ |
| $1$ | $-4$ | $(1, -4)$ |
| $2$ | $-1$ | $(2, -1)$ |
| $3$ | $2$ | $(3, 2)$ |



The graph is not symmetric about the $x$-axis (e.g. $(3, 2)$ is on the graph but $(3, -2)$ is not)

The graph is not symmetric about the $y$-axis (e.g. $(3, 2)$ is on the graph but $(-3, 2)$ is not)

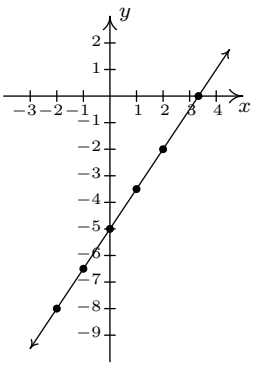
The graph is not symmetric about the origin (e.g. $(3, 2)$ is on the graph but $(-3, -2)$ is not)

48. $3x - 2y = 10$
Re-write as: $y = \frac{3x-10}{2}$.

$x$-intercepts: $\left(\frac{10}{3}, 0\right)$

$y$-intercept: $(0, -5)$

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| $-2$ | $-8$ | $(-2, -8)$ |
| $-1$ | $-\frac{13}{2}$ | $\left(-1, -\frac{13}{2}\right)$ |
| $0$ | $-5$ | $(0, -5)$ |
| $1$ | $-\frac{7}{2}$ | $\left(1, -\frac{7}{2}\right)$ |
| $2$ | $-2$ | $(2, -2)$ |



The graph is not symmetric about the $x$-axis (e.g. $(2, -2)$ is on the graph but $(2, 2)$ is not)

The graph is not symmetric about the $y$-axis (e.g. $(2, -2)$ is on the graph but $(-2, -2)$ is not)

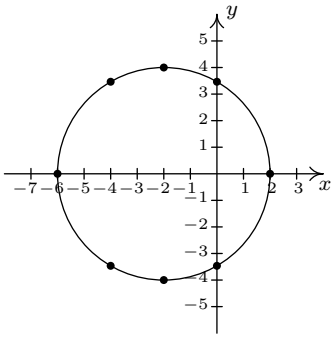
The graph is not symmetric about the origin (e.g. $(2, -2)$ is on the graph but $(-2, 2)$ is not)

49. $(x+2)^2 + y^2 = 16$
Re-write as $y = \pm\sqrt{16-(x+2)^2}$.

$x$-intercepts: $(-6,0)$, $(2,0)$

$y$-intercepts: $\left(0, \pm 2\sqrt{3}\right)$

| $x$ | $y$ | $(x,y)$ |
|---|---|---|
| $-6$ | $0$ | $(-6,0)$ |
| $-4$ | $\pm 2\sqrt{3}$ | $\left(-4, \pm 2\sqrt{3}\right)$ |
| $-2$ | $\pm 4$ | $(-2, \pm 4)$ |
| $0$ | $\pm 2\sqrt{3}$ | $\left(0, \pm 2\sqrt{3}\right)$ |
| $2$ | $0$ | $(2,0)$ |




The graph is symmetric about the $x$-axis

The graph is not symmetric about the $y$-axis (e.g. $(-6,0)$ is on the graph but $(6,0)$ is not)

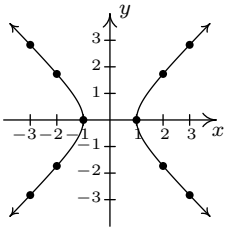
The graph is not symmetric about the origin (e.g. $(-6,0)$ is on the graph but $(6,0)$ is not)

50. $x^2 - y^2 = 1$
Re-write as: $y = \pm\sqrt{x^2-1}$.

$x$-intercepts: $(-1,0), (1,0)$

The graph has no $y$-intercepts

| $x$ | $y$ | $(x,y)$ |
|---|---|---|
| $-3$ | $\pm\sqrt{8}$ | $(-3, \pm\sqrt{8})$ |
| $-2$ | $\pm\sqrt{3}$ | $(-2, \pm\sqrt{3})$ |
| $-1$ | $0$ | $(-1,0)$ |
| $1$ | $0$ | $(1,0)$ |
| $2$ | $\pm\sqrt{3}$ | $(2, \pm\sqrt{3})$ |
| $3$ | $\pm\sqrt{8}$ | $(3, \pm\sqrt{8})$ |




The graph is symmetric about the $x$-axis

The graph is symmetric about the $y$-axis

The graph is symmetric about the origin

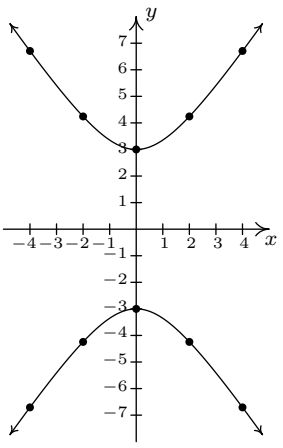
51. $4y^2 - 9x^2 = 36$

Re-write as: $y = \pm\frac{\sqrt{9x^2+36}}{2}$.

The graph has no $x$-intercepts

$y$-intercepts: $(0, \pm 3)$

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| $-4$ | $\pm 3\sqrt{5}$ | $\left(-4, \pm 3\sqrt{5}\right)$ |
| $-2$ | $\pm 3\sqrt{2}$ | $\left(-2, \pm 3\sqrt{2}\right)$ |
| $0$ | $\pm 3$ | $(0, \pm 3)$ |
| $2$ | $\pm 3\sqrt{2}$ | $\left(2, \pm 3\sqrt{2}\right)$ |
| $4$ | $\pm 3\sqrt{5}$ | $\left(4, \pm 3\sqrt{5}\right)$ |




The graph is symmetric about the $x$-axis

The graph is symmetric about the $y$-axis

The graph is symmetric about the origin

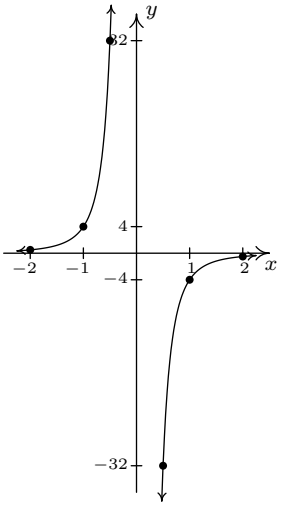
52. $x^3 y = -4$

Re-write as: $y = -\dfrac{4}{x^3}$.

The graph has no $x$-intercepts

The graph has no $y$-intercepts

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| $-2$ | $\frac{1}{2}$ | $(-2, \frac{1}{2})$ |
| $-1$ | $4$ | $(-1, 4)$ |
| $-\frac{1}{2}$ | $32$ | $(-\frac{1}{2}, 32)$ |
| $\frac{1}{2}$ | $-32$ | $(\frac{1}{2}, -32)$ |
| $1$ | $-4$ | $(1, -4)$ |
| $2$ | $-\frac{1}{2}$ | $(2, -\frac{1}{2})$ |




The graph is not symmetric about the $x$-axis (e.g. $(1, -4)$ is on the graph but $(1, 4)$ is not)

The graph is not symmetric about the $y$-axis (e.g. $(1, -4)$ is on the graph but $(-1, -4)$ is not)

The graph is symmetric about the origin